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THE FINITE ELEMENT CODE NOSA

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The finite element code NOSA (NOn-Linear Structural Analysis) [1] has been developed by the Mechanics of Materials and Structures Laboratory of the ISTI-CNR with the aim of testing new constitutive models for materials. It has moreover been applied to checking the algorithms used for integrating the equations of motion, as well as other numerical techniques for solving structural engineering problems. The development of NOSA began in 1980 and has continued over the ensuing years along the research lines of the Lab.

The first version of the code included plane, three-dimensional and axisymmetric isoparametric elements [2] and allowed for elastic and elastic-plastic analyses in the presence of infinitesimal strains with the work-hardening models described in [3]. The code has subsequently been extended to include cases of finite strains and contact problems, based on studies performed on both the constitutive equations [4 - 7] and the methods for numerical integration of the equations of motion, in the presence of follower forces [8, 9, 10].

Over recent decades, constitutive models and calculation techniques have become available that enable realistic description of the static behaviour of masonry structures. Several studies [11 - 16] have led to a better understanding of the constitutive equation of materials not withstanding tension, known in the literature as *masonry-like* or *no-tension* materials. Within this framework, masonry is modelled as a nonlinear elastic material, with zero tensile strength and infinite or bounded compressive strength.

In order to study real problems, the equilibrium problem of masonry structures can be solved via the finite element method. To this end, suitable numerical techniques have been developed [13]-[16] based on the Newton-Raphson method for solving the nonlinear system obtained by discretising the structure into finite elements. Their application requires that the derivative of the stress with respect to the strain be explicitly known, as this is needed in order to calculate the tangent stiffness matrix. The numerical method studied has therefore been implemented into the NOSA code to enable determination of the stress state and the presence of any cracking. It can moreover be applied to modelling needed restoration and reinforcement operations on constructions of particular architectural interest [17].

The code has been further enhanced to be able to perform nonlinear heat-conduction analysis on solids even in the non-stationary case, with boundary conditions concerning temperature and thermal fluxes. Today, the code provides for thermo-mechanical analysis of no-tension solids whose mechanical characteristics depend on temperature in the presence of thermal loads [18], [19], [16].

Finally, numerical solution of dynamic problems requires direct integration of the equations of motion [20]. In fact, due to the nonlinearity of the adopted constitutive equation, the mode-superposition method is meaningless. With an aim to solving such problems, we have instead implemented the Newmark [21] method in NOSA to perform the integration with respect to time of the system of ordinary differential equations obtained by discretising the structure into finite elements. Moreover, the Newton-Raphson scheme, needed to solve the nonlinear algebraic system obtained at each time step, has been adapted to the dynamic case. In the framework of this formulation, the uniqueness of the solution of the dynamic problem in not guaranteed, even in terms of stress, which on the contrary holds for the static case [16]. In order to overcome this drawback, a viscous stress depending linearly on the strain rate has been introduced, thanks to which the uniqueness of the displacement, strain and stress fields is ensured [22].

The code has been successfully applied to the analysis of arches and vaults [23], as well as some buildings of historical and architectural interest, amongst which the chimney of the Vecchi Macelli [24], the Medici Arsenal [25, 16] and the San Pietro in Vinculis Church [26] in Pisa, the San Nicolò Motherhouse in Noto [27], the Goldoni Theatre in Livorno [28], the Baptistery of the Volterra Cathedral, the bell tower of Buti [29], the church of Santa Maria Maddalena in Morano Calabro [16], the church of San Ponziano in Lucca [30], the church of Santa Maria della Roccella in Roccelletta di Borgia [31] and the Rognosa tower in San Gimignano [32, 33].
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Over the past twenty years many engineering students at the University of Pisa have collaborated on the development of the code as part of their degree or doctorate thesis preparation.

For a complete and detailed description of the theories and algorithms used in NOSA, the interested reader is referred to the following bibliography.

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